How Dice can be used to Predict Heat Flow and the Fate of the Universe.

Connor Rosenberry

Longwood University, Farmville, Virginia.

Purpose: In this experiment, we will study the properties of entropy, multiplicity and probability using dice. We will roll 4 & 8 sided dice together and 4, 6, and 8 sided dice together for many trials and we will record the sum of the dice for each roll. We will then calculate each possible outcome's expected multiplicity and probability of occurring. We can also predict each outcome's expected entropy and compare these expected values of multiplicity and entropy to their actual values we found in this experiment. In doing so, we will be able to see the natural tendency of systems to always move towards the region of maximum multiplicity and entropy.

Background:

- Entropy is a measure of disorder. For example, a shuffled deck of cards has a higher entropy than a sorted deck.
- The 2nd Law of Thermodynamics tells us that systems always head to the region of maximum disorder. In this region also lies the most number of microstates.
- A macrostate is a potential outcome of a system (e.g. rolling a total of 5 across the dice thrown). A microstate is a specific microscopic configuration of a system that tells us how many ways we can achieve a certain macrostate.
- The amount of times a certain macrostate occurs is called its multiplicity. The probability of a certain macrostate occuring is defined as the multiplicity of that macrostate divided by the total number of microstates,

$$P = W/N$$

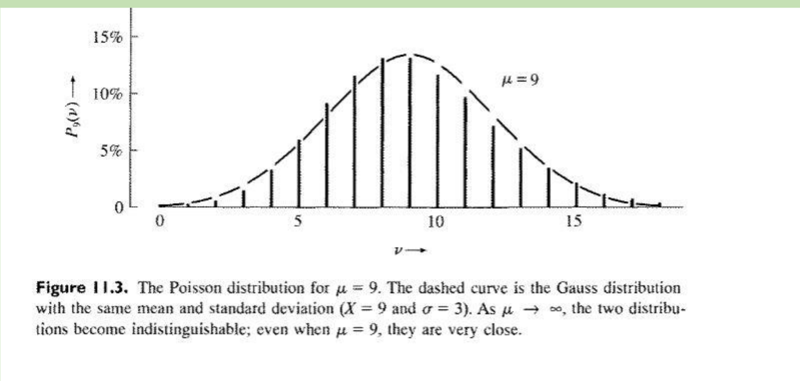
where P is the probability of the macrostate occuring, W is the multiplicity of that macrostate, and N is the total number of microstates. Entropy is also related to multiplicity,

$$S = K_b \ln(W)$$

where S is entropy in (J/K), K_b is Boltzmann's constant, and ln(W) is the natural log of a certain macrostate's multiplicity.

Probability Distributions:

- Poisson Distribution- This distribution describes the results of experiments in which we count events that occur at random but have an average rate at which they occur.
- Normal Distribution- The standard bell curve that is dependent on the mean and standard deviation. The sample cannot be perfectly distributed if it contains a finite number of observations.

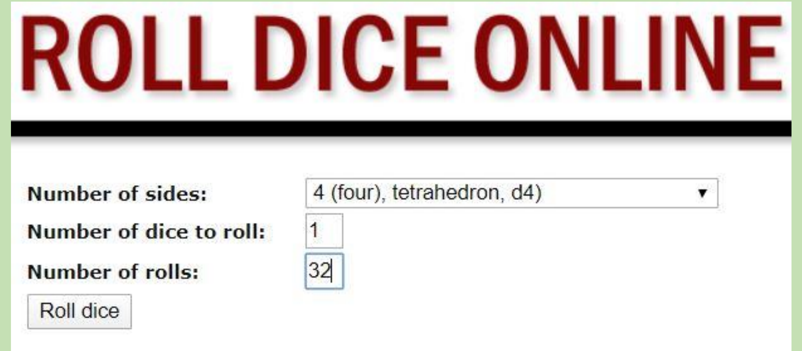


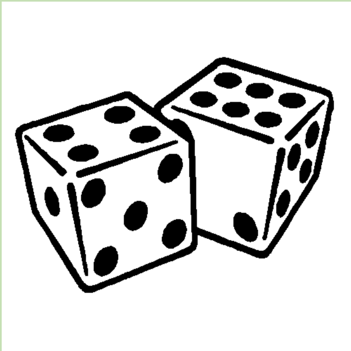
- Figure 11.3[1] to the left shows the differences and similarities between the Normal, Gauss, Distribution and the Poisson Distribution.
- The Poisson Distribution is given by $P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}$ where μ is the expected mean count of a certain macrostate's multiplicity and ν is the multiplicity of a certain macrostate for any roll of the dice.

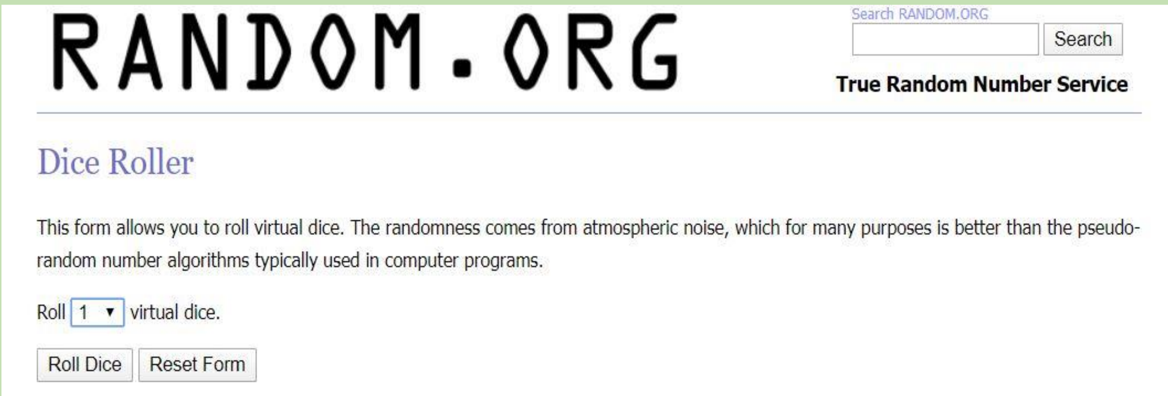
Experimental Procedure: We rolled 4 and 8 sided dice together for 32 rolls and then 128 rolls using roll-dice-online.com. We recorded the outcome on each dice to get the macrostate. Then we calculated the expected entropy and expected multiplicity of each possible macrostate and we compared them to their actual experimental values. Then we repeated these same steps for 4, 6, and 8 sided dice rolled together for 192 and 384 rolls. For the 6 sided dice, we used random.org to roll the dice which uses atmospheric noise to generate randomness.

Dice Roll	Probability	Expected Multiplicity	Expected Entropy (J/K)	Actual Multiplicity	Actual Entropy (J/K)
2	1/32	1	0	2	9.57236E-24
3	1/16	2	9.57236E-24	0	Undefined
4	3/32	3	1.51718E-23	2	9.57236E-24
5	1/8	4	1.91447E-23	10	3.17987E-23
6	1/8	4	1.91447E-23	5	2.22263E-23
7	1/8	4	1.91447E-23	1	0
8	1/8	4	1.91447E-23	3	1.51718E-23
9	1/8	4	1.91447E-23	3	1.51718E-23
10	3/32	3	1.51718E-23	2	9.57236E-24
11	1/16	2	9.57236E-24	3	1.51718E-23
12	1/32	1	0	1	0

The table above shows the probability, expected multiplicity and entropy, and actual multiplicity of each possible macrostate when we rolled 4 and 8 sided dice together for 32 rolls. From this table we can see how we are most likely to roll a 5 through a 9 in total across the two dice. We can also see that is where the system is most disordered.

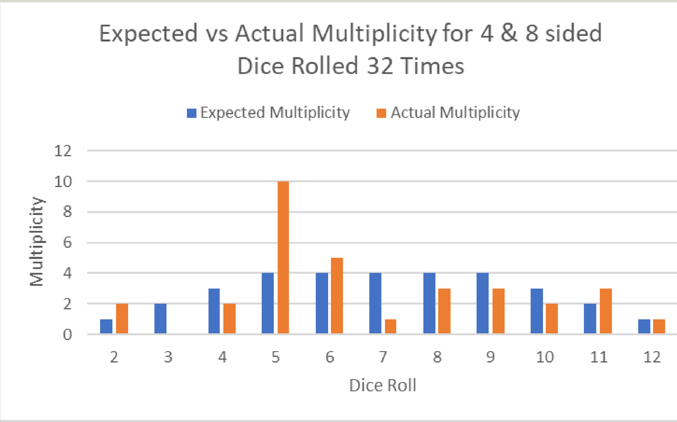


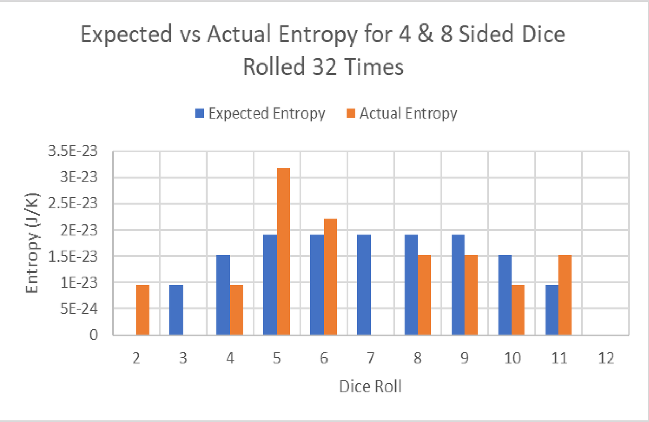


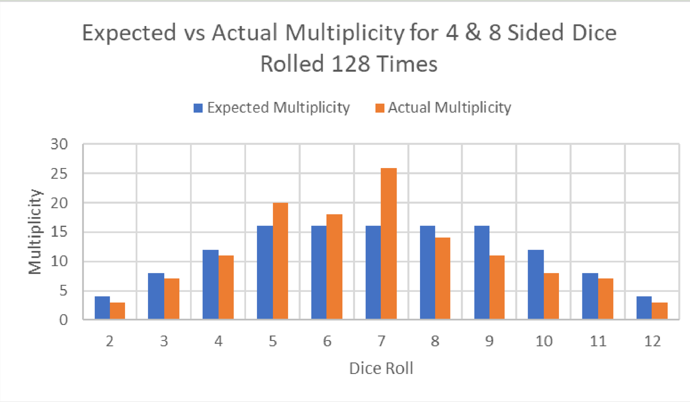


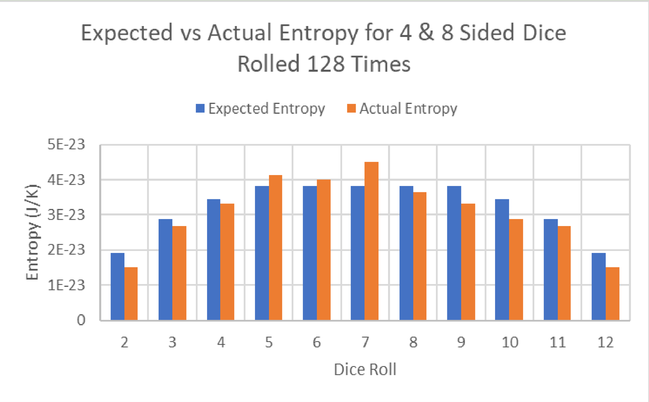
Error Analysis:

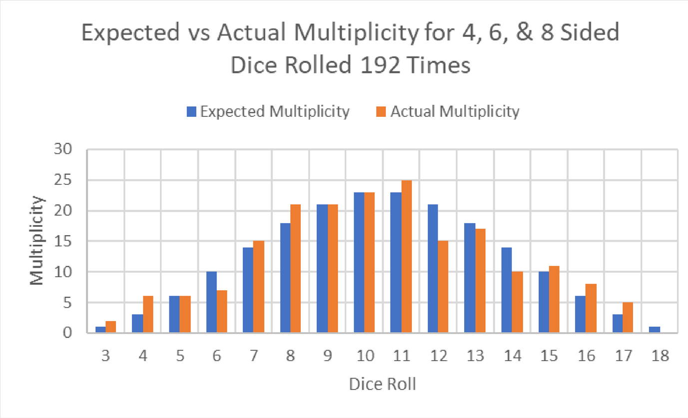
- Shown above are the online dice simulators we used to roll our dice. It would have been ideal to use actual dice, but since we did not own any 4 or 8 sided dice we had to use an online dice roller. To make it consistent, we decided to use an online dice simulator for all of our dice rolls. Unfortunately, the online dice rollers may not have been perfectly random.
- We calculated the standard deviation and standard deviation of the mean for the expected and actual multiplicity and entropy. Which told us how far we could expect a multiplicity of a certain macrostate to vary in between dice rolls.
- The following histograms will show the relationship between expected and actual multiplicity and entropy. We will be able to see nature's tendency to move in the direction of the most disordered states and see why so many things move in one direction only.

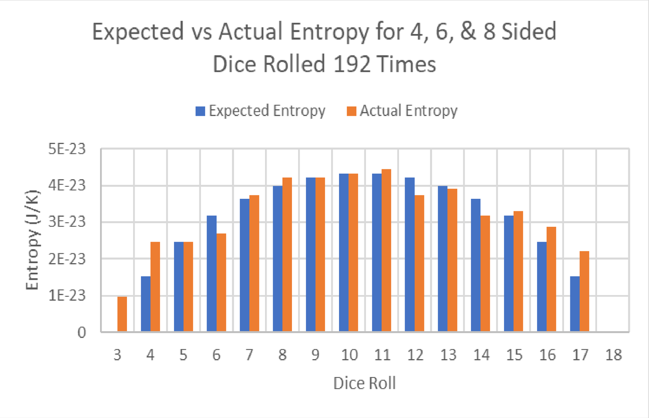


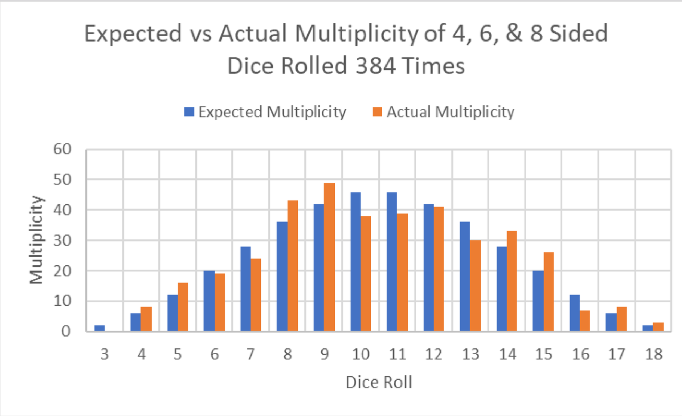


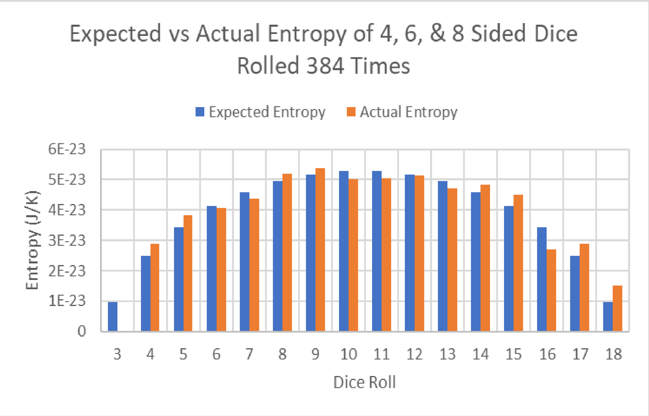












Conclusion: From these histograms, we should be able to see the most probable states we found the dice to be in. The system is most likely to be found where the most microstates are. In other words, where all the possibilities are. There were some anomalies but we are still able to see the natural tendency of systems to move in the direction of maximum entropy and multiplicity. This experiment shows how systems always travel in one direction and entropy always increases over time. Thermodynamics tell us that heat is most likely to flow from high to low. This is why we put ice in our drink when we want to cool it down. The heat from the liquid travels to the ice where it is absorbed and cools the drink. However, heat does not have to flow in this direction and we can see that in this experiment. Ice could actually become colder while warming your drink. The only thing preventing this from happening is extremely unlikely odds. Physicists also use the concept of the "Arrow of Time" which means that the entropy of the universe always increases over time, to predict the fate of the universe. We predict the universe will end in a "Big Freeze" as galaxies continue to move away from one another. We can see this "Arrow of Time" in our histograms above. As we approach the peak of the graph from the left and from the right, we notice that entropy is always increasing or not changing. The probability of moving away from this peak is astronomically small on a large scale such as the universe. That is why we say the entropy of the universe can never decrease.

References: [1]"An Introduction to Error Analysis" John R. Taylor.