Statistical Mechanics and Thermodynamics Shown with Dice

University Physics II Lab – PHYS 223

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Purpose

In this lab, we will seek to demonstrate the Second Law of Thermodynamics by performing a series of dice experiments showing the tendency towards disorder and its correlation to probabilities.

Background and Theory

Entropy is a fascinating concept in the fields of Thermodynamics and Statistical Mechanics and the focus of this experiment.

- Entropy is a quantitative measurement of the disorder of a system. [3]
- The Second Law of Thermodynamics states that within an isolated system the total value of entropy will never decrease. [3]
- A system is isolated if it does not interact with anything outside the system to send or receive energy.
- If we view a system as a collection of smaller, identical, and independent systems then we have what statistical mechanics calls an ensemble.
- An ensemble can exist in a variety of states called macrostates, where the whole system can be given one value.
- Each arrangement of the system that can form a given macrostate is called a microstate.
- The number of possible macrostates that an ensemble can have will always be equal to or greater than the number of microstates.
- The number of microstates that can form a given macrostate is called the multiplicity of that microstate for that system.

The multiplicity can be related to the probability, or relative frequency of occurrence, P, with the equation

$$P = W/N \qquad (1)$$

where W is the multiplicity, and N is the total number of microstates that occur or are measured. Using the multiplicity, we can calculate the entropy within a system with the equation

$$S = k_B \ln W \qquad (2)$$

where S is the entropy, W is the multiplicity, and k_B is Boltzmann's constant (1.380649 x 10-23 J/K).

Experiment

For this experiment we will be using dice to simulate the randomly occurring microstates within a system. Each dice roll will resemble a single microstate, with the sum of the dice being a macrostate. For a given macrostate there will be a number of possible combinations of the dice to produce the same value, this number represents the multiplicity of that macrostate. By counting the number of different combinations manually, we can find an expected multiplicity for a set number of dice rolls. By plugging the expected multiplicity into Equation (2) we can calculate an estimated value for the entropy of the system at that state.

By rolling the dice for the set number of times we can also find an actual multiplicity by counting the number of times each macrostate actually occurs. Using this actual multiplicity to calculate an actual entropy, we can compare our expected values with the experimental values to test the statistical mechanical approach to this dice system.

To find the multiplicity for a set of dice used in this experiment, we created tables like the following to count the number of times each microstate occurs.

Value Rolled		4-Sided Die			
		1	2	3	4
8-Sided Die	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8
	5	6	7	8	9
	6	7	8	9	10
	7	8	9	10	11
	8	9	10	11	12

Table 1: Possible outcomes of rolling a 4-sided die with an 8-sided die.

Since the data is found by taking a random sampling of values (dice rolls), the uncertainty in the average multiplicity was found using the statistical method and recorded as the standard error (an example can be seen in Table 2 below for Experiment part 1).

Results and Discussion

In this experiment, we used two separate dice rolling websites[1,2] to generate random data to allow us to test the second law of thermodynamics using statistical mechanics and Boltzmann's equation. We took four separate collections of data using three types of dice. The first two sets of data used 4-sided and 8-sided dice which were rolled 32 times and then 128 times. The second two sets of data used 4-sided, 6-sided, and 8-sided dice which were then rolled 192 times and then 384 times. Using the data from these rolls we were able to calculate the expected and actual values for the multiplicity and entropy related to each macrostate, and then average our results to show how the actual compares with the expected. Some of the results from our calculations are shown in the following table:

Dice Roll (Macrostate)	Probability P = W/N	Expected Multiplicity, W_E	Actual Multiplicity, W_A	Estimated Entropy, S_E [x10^{-23} J/K]	Actual Entropy, S_A [x10^{-23} J/K]
2	1/32	1	1	0	0
3	1/16	2	0	0.96	Undefined
4	3/32	3	4	1.52	1.91
5	1/8	4	2	1.91	0.96
6	1/8	4	5	1.91	2.22
7	1/8	4	5	1.91	2.22
8	1/8	4	4	1.91	1.91
9	1/8	4	4	1.91	1.91
10	3/32	3	4	1.52	1.91
11	1/16	2	3	0.96	1.52
12	1/32	1	0	0	Undefined
Mean, W_{AVG}		2.9	2.9		
Standard Deviation, σ		1.2	1.9		
Standard Error, SE(W)		0.4	0.6		

Table 2: Results from Experiment part 1

The following figures show some of the histograms for a visual comparison of the expected values with the actual values found for the multiplicity and entropy.

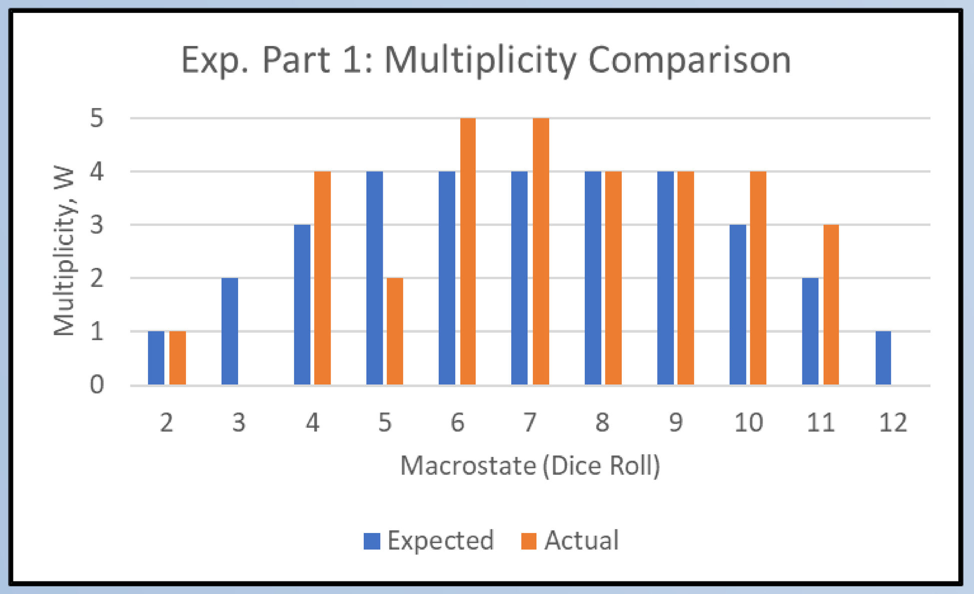

Figure 1: Histogram comparing Actual and Expected Multiplicities for Experiment part 1

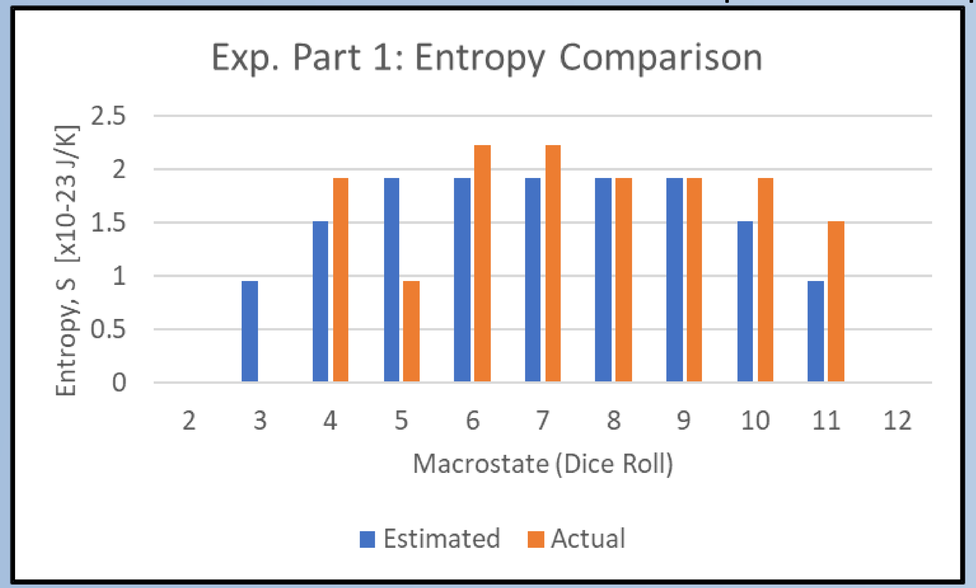

Figure 2: Histogram comparing Actual and Estimated Entropies for Experiment part 1

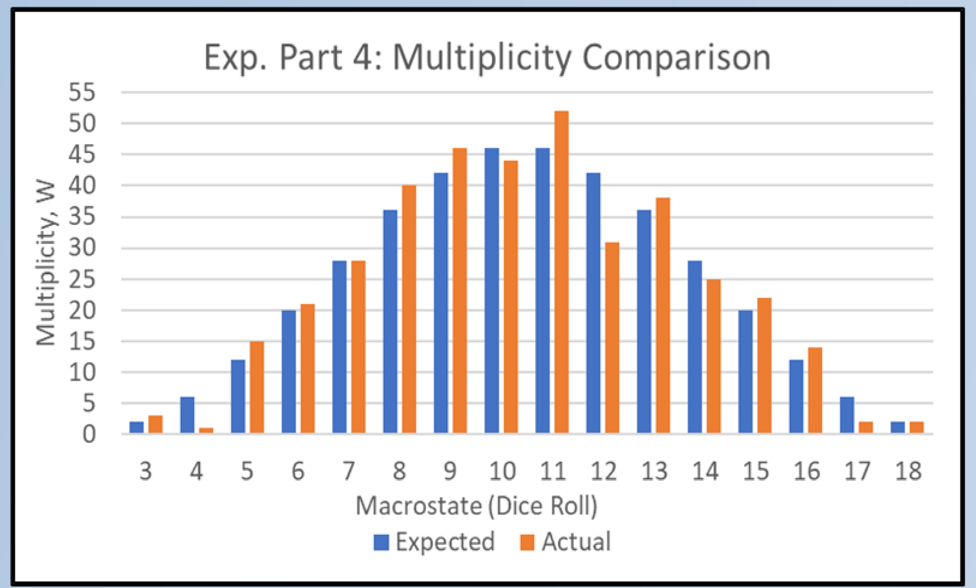

Figure 3: Histogram comparing Actual and Expected Multiplicities for Experiment part 4

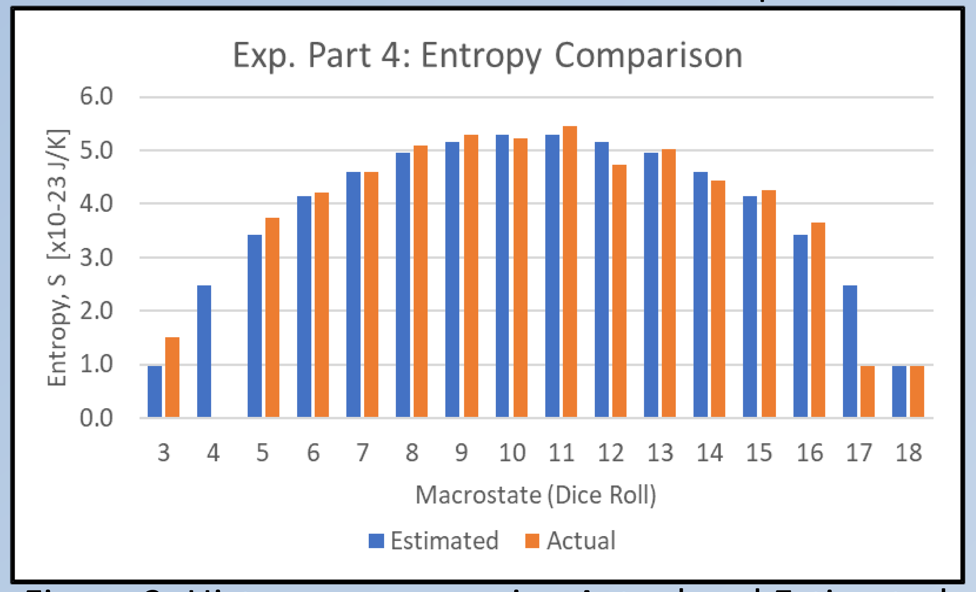

Figure 2: Histogram comparing Actual and Estimated Entropies for Experiment part 1

References: [1] www.random.org, [2] www.roll-dice-online.com, [3] Dusto, A. (2019, December 5). Third Law of Thermodynamics: Definition, Equation & Examples. Retrieved from https://sciencing.com/third-law-of-thermodynamics-definition-equation-examples-13722779.html