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Non-Local Approximation

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Recommended Citation

Hammond, Iris, "Non-Local Approximation" (2020). Fall Showcase for Research and Creative Inquiry. 91. [https://digitalcommons.longwood.edu/rci_fall/91](https://digitalcommons.longwood.edu/rci_fall/91?utm_source=digitalcommons.longwood.edu%2Frci_fall%2F91&utm_medium=PDF&utm_campaign=PDFCoverPages)

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Non-Local Approximation

Iris Hammond & Dr. Jeff Ledford

This project is a continuation of work previously done by Dr. Jeff Ledford. This research extends the non-local approximation schemes found in [1,2,3]. We seek approximands of the form $C_1 \phi(x - x_0) + C_2 \phi(x - x_1) + \cdots + C_n \phi(x - x_{n-1})$ which approximate continuous functions uniformly on closed intervals and interpolate the data $\{(x_j, y_j) : j = 0, 1, ..., n-1\}$. Our function ϕ is taken to be the (general) multiquadric

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Introduction

 $\phi(x) = (1 + x^2)$ −3/2 pictured below. This research is still ongoing.

splitting our problem into two cases, depending on the parity of n , allows us to use a Lemma 1 in [2] to show that $a_n \neq 0$. When *n* is odd, we can reduce the expression to

which is an Mth degree polynomial, whose leading coefficient is 2 ^M. Now we may use Lemma 1 in [2] which provides,

Methods

We first expand ϕ (x-y) in a Taylor series in $\frac{1}{n^2}$ y^n , this yields coefficient polynomials $A_n(x)$. If the set of these polynomials span the space of all polynomials, $\Pi[x]$, then the uniform approximation property will follow from the Stone-Weierstrass theorem. A sufficient condition for spanning $\Pi[x]$ involves showing that the leading coefficients of $A_n(x)$ are non-zero.

Lemma 1. For $N \in \mathbb{N}$, $0 \le l \le N$, and p a polynomial of degree l. We have,

The leading coefficients seems to be *aⁿ =* $n + 2$ 2 showing this pattern was then sought. We have

. The proof

which is the desired result when $n = 2M + 1$. The even case is virtually nearly identical.

To clean up this argument, we extended Lemma 1 in [2] to include polynomials of degree $M+1$.

Lemma 2. Suppose $M \in \mathbb{N}$, then we have

$$
a_n = (-2)^{-n} \sum_{j=\lceil \frac{n}{2} \rceil}^{n} 4^j \binom{-3/2}{j} \binom{j}{2j-n}
$$

$$
_{n}\Big).
$$

$$
-1)
$$

Proof. We calculate this directly from the symmetry in the binomial coefficients. Let $M=1,2,3,...$ and consider the sum

$$
a_{2M+1} = \frac{(-1)^{M+1}}{2^M M!} {2M+3 \choose 2} \sum_{j=0}^M (-1)^j {M \choose j} Q(j)
$$

where

$$
Q(x) = \frac{(2x+1)(2x+3)\cdots(2x+2M+1)}{(M+1)(2M+1)}
$$

where we reindexed, then factored out (-1) to arrive at the last equality. Now adding these expressions together and using Lemma 1 in [2] yield

$$
\sum_{j=0}^{N} (-1)^{j} {N \choose j} p(j) = \begin{cases} 0 & 0 \le l < N \\ (-1)^{N} a_N \cdot N! & l = N \end{cases}
$$

where a_N is the leading coefficient of p.

$$
(-1)^{M+1} 2^M M! \frac{(-1)^M}{2^M M!} {2M + 3 \choose 2} = {2M + 3 \choose 2}
$$

 $M + 1$ 2

$$
\sum_{j=0}^{M} (-1)^j {M \choose j} j^{M+1} = (-1)^M M! {M \choose 2}
$$

+ 1

$$
\sum_{j=0}^{M} (-1)^{j} {M \choose j} j^{M+1} = \sum_{j=0}^{M} (-1)^{j} {M \choose M-j} j^{M+1}
$$
; let $k=M-j$

Proof

References

[1] Ledford, J. Approximating continuous functions with scattered translates of the Poisson kernel, *Missouri Journal of Mathematical Sciences.* **26** (2014), no. 1, 64–69 [2] J. Ledford, On the density of scattered translates of the general multiquadratic in C([a,b]). *New York J. Math.* **20** (2014), 145–151. [3] M.J.D. Powell, Univariate multiquadric interpolation: Reproduction of linear polynomials, in *Multivariate Approximation and Interpolation (Duisberg 1989)*, Internat. Ser. Numer. Math. **94,** 227-240, Birkhäuser, Basel, 1990. □

 $(M-k)^{M+1}$

 $(k - M)^{M+1}$

 $+1$

 (m) ^{$m+1$}]

 j \bigwedge^{M} \dot{J} \dot{J} $M-1 + \cdots$ $-0 + 0 - \cdots$

0

−1

 $=$ \sum

$$
\angle_{k=M} (k)^{(M-k)}
$$

= $(-1)^{2M+1}$
$$
\sum_{k=0}^{M} (-1)^k {M \choose k} (k - k)
$$

= $-\sum_{k=0}^{M} (-1)^k {M \choose k} (k - k)^{M+1}$

 $M-k$ $\left(\overline{M}\right)$

We used Maple to generate the first few coefficient polynomials.

1, 3 x
\n2,
$$
6x^2 - \frac{3}{2}
$$

\n3, $10x^3 - \frac{15}{2}x$
\n4, $15x^4 - \frac{45}{2}x^2 + \frac{15}{8}$
\n5, $21x^5 - \frac{105}{2}x^3 + \frac{105}{8}x$
\n6, $28x^6 - 105x^4 + \frac{105}{2}x^2 - \frac{35}{16}$
\n7, $36x^7 - 189x^5 + \frac{315}{2}x^3 - \frac{315}{16}x$
\n8, $45x^8 - 315x^6 + \frac{1575}{4}x^4 - \frac{1575}{16}x^2 + \frac{315}{128}$

$$
2\sum_{j=0}^{M}(-1)^{j} {M \choose j} j^{M+1}
$$

=
$$
\sum_{j=0}^{M}(-1)^{j} {M \choose j} [j^{M+1} - (j - j - j + (j - j - j)]
$$

=
$$
\sum_{j=0}^{M}(-1)^{j} {M \choose j} [(M + 1)j^{M}]
$$

=
$$
M(M + 1) \sum_{j=0}^{M}(-1)^{j} {M \choose j} j^{M}
$$

$$
= 2 {M + 1 \choose 2} ((-1)^{M} M!) \n\therefore \sum_{j=0}^{M} (-1)^{j} {M \choose j} j^{M+1} = (-1)^{M} M!
$$

$$
-M^{2}\binom{M+1}{2}\sum_{j=0}^{M}(-1)^{j}
$$

= $2\binom{M+1}{2}\sum_{j=0}^{M}(-1)^{j}\binom{M}{j}j^{M}$